CP Violating Phases and their Implications on Leptogenesis

Anupam Yadav¹, Sabeeha Naaz², Jyotsna Singh³, R.B. Singh⁴

¹,²,³,⁴Department of Physics, University of Lucknow, Lucknow-226007, India

Abstract - We have developed a framework to predict the conditions required for successful leptogenesis. This framework is based on minimal see-saw model. This work relates the high energy leptogenesis parameters to the low energy neutrino oscillation parameters, which are measurable at low energy experiments. With the help of known low energy parameters, we have tried to impose constraints on the leptogenesis parameters i.e, bound on the mass of heavy right handed neutrino and phases (majorana (ψ1) and ψCP).

Key words : Leptogenesis, Minimal See-Saw Model, CP Violation, Low Energy Parameters and heavy right handed neutrinos.

I. INTRODUCTION

Even in the absence of any indication of the new physics from the collider experiments, there are enough phenomenological grounds to develop a physics which looks beyond the standard model (SM) i.e. to explain neutrino masses and mixing, origin of matter-antimatter symmetry of the universe and insight for understanding the dark matter (DM). Here we discuss the extension in the SM which can reasonably address the three compelling evidences of the new physics in a unified picture. This extension in SM is made by the introduction of right handed (RH) neutrinos with Yukawa couplings, few additional couplings to the Higgs and a majorana mass term. Here in this work we focus on the physics which can explain the reason behind the prevailing matter-antimatter asymmetry in the universe. Various attempts have been made to explain the origin of baryon asymmetry, amongst those the leptogenesis proposed by Fukugita and Yanagida[1] has gathered much attention because it can be realized in the context of see-saw model, which was proposed to explain the tiny masses of neutrinos.

To perceive leptogenesis, new sources of CP violation are required and hence it will be interesting to investigate whether low energy CP phases have any impact on resolving the problem of leptogenesis or not.

The purpose of this work is to explore how leptogenesis can be related to the low-energy CP phases and establish a relationship between low and high energy parameters.

In an attempt to make a quantitative analysis of the connection between low energy parameters and leptogenesis (high energy), we depend on the minimal CP violating see-saw model (MSM) which consist of two heavy majorana neutrinos along with three active left handed neutrinos[2][3][4]. Leptogenesis is sensitive to the lightest majorana mass of the lightest majorana particle in a scenario where heavy majorana mass follows hierarchical pattern and CP violating phases.

II. LEPTOGENESIS AND MINIMAL SEE-SAW MODEL:

The Lagrangian for the leptonic sector of the minimal see-saw model, in a basis where both heavy majorana and charged lepton mass matrices are real and diagonal is given by [5][6]:

\[ L = -\bar{l}_L m_{il} l^c_R - \nu_L m_{Dij} N_{Rj} - \frac{1}{2} N_{Ri}^c M_{ij} N_{Rj} \]

(1)

where i= 1,2,3 (light neutrinos), j=1,2 and mD is 3x2 complex Dirac mass matrix, NR is right handed neutrino and IL and IR are left and right handed lepton respectively. Minimum two right handed neutrinos along with three SU(2) doublets are required to break the CP symmetry in the see-saw model. From the see-saw mechanism we know that effective light neutrino mass matrix can be expressed as :

\[ m_{\text{eff}} = m_D \frac{1}{M} m_D^T \]

(2)

where M= diag[M1,M2]. The matrix meff can be diagonalized by 3 × 3 PMNS matrix [7] can be expressed as :

\[ m_{\text{eff}} = \tilde{U}_{\text{PMNS}} m_{\nu} \tilde{U}_{\text{PMNS}}^T \]

(3)

where
\[ \tilde{U} = R(\theta_{23})R(\theta_{13})R(\theta_{12})P(\phi_1,\phi_2) \]  
(4)

\[ P = \text{diag}(1, e^{i\phi_1/2}, e^{i\phi_2/2}) \]

The Majorana phases can be taken along with the diagonal mass matrix and then the meff can be give as :

\[ m_{\text{eff}} = U \text{diag}(m_1,\hat{m}_2, \hat{m}_3) U^T \]  
(5)

where

\[ U = \hat{U} P^{-1}, \hat{m}_2 = m_2 e^{i\phi_1} \text{and} \hat{m}_3 = m_3 e^{i\phi_2} \]

In our work we consider U to be UTB (Tri bimaximal mixing) [8] and this selection turns the effective neutrino mass to be:

\[ m_{\text{eff}} = m_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \frac{\hat{m}_2 - m_1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \frac{\hat{m}_3 - m_1}{2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \]  
(6)

The minimal model can be obtained with UTH and by assuming zero mass to one generation of neutrinos in to hierarchies i.e. Normal Hierachy (NH), \( m_1 \leq m_2 < m_3 \) and Inverted Hierachy (IH), \( m_3 < m_1 \leq m_2 \), the following mass eigenvalues can be obtained :

For NH :

\[ m_1 = 0 \]

\[ m_2 = \sqrt{m_2^2 - m_1^2 + \Delta m_{\text{sol}}^2} \]  
(7)

\[ m_3 = \sqrt{m_3^2 - m_1^2 + \Delta m_{\text{atm}}^2} = \sqrt{\Delta m_{\text{sol}}^2 + \Delta m_{\text{atm}}^2} \]

For IH :\n
\[ m_1 = \sqrt{\Delta m_{\text{atm}}^2 - \Delta m_{\text{sol}}^2} \]  
(8)

\[ m_2 = \sqrt{\Delta m_{\text{atm}}^2}, \]

\[ m_3 = 0 \]

In both the cases phase transformation \( P = \text{diag}(1, e^{i\phi_1/2}, 1) \) is used. In this way we have to examine \( \hat{m}_2 \) only in order to investigate CP violating contribution of Majorana phase.

The Dirac mass matrix of eq. (2) can be expressed as :

\[ m_D = \begin{bmatrix} \sqrt{M_1 a_1} & \sqrt{M_2 b_1} \\ \sqrt{M_1 a_2} & \sqrt{M_2 a_2} \\ \sqrt{M_1 a_3} & \sqrt{M_2 b_3} \end{bmatrix} \]  
(9)

where

\[ a_1 = \sqrt{m_{11} - b_1^2}, b_1 = \sqrt{m_{11} - a_1^2} \]  
(10)

\[ a_i = \frac{1}{m_{11}}[a_i m_{ii} - \sigma_i b_1 \sqrt{m_{11} m_{ii} - m_{ii}^2}] \]  
(11)

\[ b_i = \frac{1}{m_{11}}[b_i m_{ii} + \sigma_i a_1 \sqrt{m_{11} m_{ii} - m_{ii}^2}] \]  
(12)
where \( m_{ij} \) are the elements of \( \text{meff} \) and \( \text{si} \) a sign +. Since mass matrix \( mD \) is a 3×2 matrix, there are 6 cases for texture zero. In this work we are considering a case where \( a2 = 0 \). For both hierarchies this can be illustrated as:

For NH:
\[ a2 = 0, a1 \neq b1, ai \neq in \text{NH case where } b2 = 0 \]  

and for IH:
\[ a2 = 0, a3 = 0, a1 \neq b1, ai \neq bi \text{ in NH case where } b2 = 0, b3 = 0 \]

### III. BARYOGENESIS VIA LEPTOGENESIS:

The baryon asymmetry observed in universe [9] [10] can be given as:

\[ Y_B = \frac{n_B - n_{\bar{B}}}{s} \approx \left(8.8 - 9.8\right) \times 10^{-11} \]

where \( n_B \) and \( n_{\bar{B}} \) are the density of baryons and anti baryons and \( s \) is the entropy. The produced baryon asymmetry can be related to the lepton asymmetry through sphaleron process [11]

\[ Y_B = a \frac{Y_{B-L}}{a-1} \]

\[ a = \frac{8N_f}{22N_f + 13N_H}, \frac{N_f}{N_H} \]

where \( N_f \) is the generations of fermions and \( N_H \) is number of Higgs doublets. In SM \( NH = 1 \) which gives \( a = 28/79 \). The see-saw mechanism can generate sufficient lepton asymmetry for the observed baryon asymmetry. The necessary ingredients for the generation of lepton asymmetry are production of CP asymmetry and out of equilibrium condition. Then the lepton asymmetry, \( Y_L \) can be parametrized by two factors \( \epsilon_i \), the size of CP asymmetry and \( k \), the dilution factor from wash out processes.

\[ Y_L = \frac{n_L - n_{\bar{L}}}{s} = \frac{\epsilon_i k}{g^*} \]

where \( g^* \neq 110 \) is the number of degree of freedom and \( n_L \) and \( n_{\bar{L}} \) are the density of leptons and anti leptons.

The CP asymmetry \( \epsilon_i \) produced by the decays of heavy Majorana neutrinos [12] is given as:

\[ \epsilon_i = \frac{\Gamma(N_i \to lH) - \Gamma(N_i \to \bar{l}H^*)}{\Gamma(N_i \to lH) + \Gamma(N_i \to \bar{l}H^*)} \]

Here subscript \( i \) denotes the generation. When two generations of heavy neutrinos have hierarchical structure, i.e, \( M1 < M2 \), then \( \epsilon_i \) in eq. (19) will be obtained by the decay of \( M1 \) [13][14]. In this case:

\[ \epsilon_i = \frac{1}{8\pi v^2} \text{Im} \left[ \left( m^\dagger D m_D \right)^2 \right]_{12} f \left( \frac{M_2}{M_1} \right) \]

\[ f \left( \frac{M_2}{m_1} \right) \]

where \( v = 174 \text{GeV} \) and \( f \left( \frac{M_2}{m_1} \right) \) represents vertex and self energy contributions to the decay width. This loop contribution in SM can be given as:

\[ f \left( x \right) = x \left[ 1 - \left( 1 + x^2 \right) \ln \frac{1 + x^2}{1 - x^2} + \frac{1}{1 - x^2} \right] \]
For large value of x, f(x) is \((-3/2)x-1\). From eq. (20) we take out the factors which depends on the dirac matrix:

\[
\frac{\text{Im}[ (m_D^\dagger m_D)_{12}^2 ]}{(m_D^\dagger m_D)_{11}} = M_2 \frac{\text{Im}[ (a_1^*b_1 + a_2^*b_2 + a_3^*b_3)^2 ]}{|a_1|^2 + |a_2|^2 + |a_3|^2} = M_2 \Delta_i
\]

For case M2 >> M1, the CP asymmetry of eq. (20) reduces to:

\[
\epsilon_1 \approx \frac{3}{16\pi v^2} M_1 \Delta_i
\]

For the case considered in our analysis, a2 = 0, the CP asymmetry can be expressed as:

\[
\epsilon_1 \propto c_{12}^2 c_{23}^2 \sin 2\theta - 2c_{12} s_{12} c_{23} s_{13} \sin(\delta_{CP} + 2\phi)
\]

For both, NH and IH. Along with mixing angles, \( \Delta \) in itself contains mass terms also.
The dilution factor \( k \) appearing in eq. (18) can be simply parametrized in terms of \( K \) defined as:

\[
K = \frac{\Gamma_1}{H}
\]

where \( \Gamma_1 \) is the tree level decay width of N1 and H is the Hubble parameter at temperature M1, when \( K=\Gamma_1/H < 1 \) describes out of thermal equilibrium processes and \( \Delta < 1 \) illustrates wash out effects [15].

\[
\kappa \approx \frac{0.3}{K (\log K)^{0.6}}, \text{for } 10 < K < 10^6
\]

\[
\kappa = \frac{1}{2\sqrt{2}+9}, \text{for } 0 < k < 10,
\]

\[
\Gamma_1 = \frac{(m_D^\dagger m_D)_{11} M_1}{8\pi v^2} \text{ and } H = 1.66 g_*^{1/2} T^2 / M_{pl}
\]

substituting in eq. (25), at temperature \( T = M_1 \), this ratio can be given as:

\[
K = \frac{M_{pl}}{1.66 \sqrt{g_*} 8\pi v^2} \frac{(m_D^\dagger m_D)_{11}}{M_1}
\]

using dirac matrix expressed in eq. (9), we get:

\[
K \approx \frac{1}{10^{-3} eV} \left( |a_1|^2 + |a_2|^2 + |a_3|^2 \right)
\]

for the case considered here i.e. a2 = 0:

\[
K(\text{NH}) \approx \frac{(2m_2^2 + 3m_3^2)}{\left(10^{-3} eV\right)\sqrt{4m_2^2 + 9m_3^2 + 12m_2m_3 \cos \phi}}
\]

\[
K(\text{IH}) \approx \frac{(m_1^2 + 2m_3^2)}{\left(10^{-3} eV\right)\sqrt{m_1^2 + 4m_2^2 + 4m_1m_2 \cos \phi}}
\]

for the selected case keeping these value in eq. (26) and (27), we can compute \( \Box \).
IV. NUMERICAL RESULTS

For our numerical analysis, the low energy parameters inputs are taken from the current experimental data given in [16][17] [18], and is expressed in table (I). For the active three neutrino mixing angles we have considered the best fit value.

Table 1 : The value of various selected parameters used for our analysis

<table>
<thead>
<tr>
<th>¹2 ¹3 ²3 CP</th>
<th>Δm₃² (10⁻⁵ eV²)</th>
<th>Δm₃₁ (10⁻³ eV²)</th>
<th>M²(GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.55</td>
<td>2.50</td>
<td>8.7×10¹¹</td>
<td></td>
</tr>
</tbody>
</table>
The value of $Y_R$ considered in our analysis [19] :

$$Y_B = \left(8.65 \pm 0.085\right) \times 10^{-11} \quad (32)$$

The analysis is provided for the independent parameters $M_1$, $\phi_1$ and $\phi_{CP}$ in the limit of $M_2 >> M_1$.

For the baryogenesis parameters correlation we have quantified the value of CP asymmetry $Y_R$ and dilution factor from the wash out processes $k$.

The fig. (1) of our analysis indicates the allowed parameter space for lightest right handed neutrino mass $M_1$ and low energy dirac CP phase. In this analysis we can perceive that with proper selection of $M_1$ and $\phi_{CP}$, we can observe the baryogenesis even if the majorana phase is zero. Different papers have stated that the lowest limit of $M_1 = 109$GeV for the observance of baryogenesis. Looking at fig. (1) we can say that favourable $\phi_{CP}$ range, when majorana phase is considered to be zero is $150\degree - 250\degree$. This range of values falls in the range of $\phi_{CP}$ predicted by NoVA and T2K experiments [16].

The same analysis with majorana phase $84.5\degree$ predicts that roughly all $\phi_{CP}$ values are allowed for the observance of baryogenesis. But a careful selection of parameter space is required as different values of $\phi_{CP}$, advocates different values of $\phi_{1}$ CP. The parameter space for NH and IH are roughly same.

Fig.(2) of our analysis shows the allowed region of parameter space of $M_1$ and majorana phase $\phi_1$ for different values of $\phi_{CP}$. This analysis is performed for both the hierarchies, NH and IH. If $M_1 = 109$GeV is enough to produce baryogenesis then roughly all values of majorana phase is allowed with $\phi_{CP} = 228\degree$. The favourable range of $\phi_1$ is $60\degree$ to $90\degree$ and $220\degree$ to $260\degree$.

V. CONCLUSIONS

The origin of the matter antimatter asymmetry of the universe is one of the greatest mysteries in physics. In this work we have investigated if low energy CP phase ($\phi_{CP}$) can be responsible for the observed baryon asymmetry of the universe. For a certain texture of the dirac mass matrix the allowed regions of leptogenesis is checked, for different parameters involved in the origin of matter antimatter asymmetry. In the limit of $M_2 >> M_1$ the leptogenesis depends on the two low energy CP phases and the mass $M_1$ of lightest right handed neutrinos. By applying the experimental bounds of matter antimatter asymmetry, we have checked the allowed range of parameter space ($\phi_{1}$, $M_1$) for fixed value of $\phi_{CP}$ and ($\phi_{CP}$, $M_1$) for fixed value of $\phi_{1}$.

VI. REFERENCES